A New Construction of 16-QAM Golay Complementary Sequences

Chan Vee Chong, Raman Venkataramani, *Member, IEEE*, and Vahid Tarokh, *Member, IEEE*

***Abstract*—We present a new construction of 16-QAM Golay sequences of length $n = 2^m$. The number of constructed sequences is $(14 + 12m)(m!/2)4^{m+1}$. When employed as a code in an orthogonal frequency-division multiplexing (OFDM) system; this set of sequences has a peak-to-mean envelope power ratio (PMEPR) of 3.6. By considering two specific subsets of these sequences, we obtain new codes with PMEPR bounds of 2.0 and 2.8 and respective code sizes of $(2 + 2m)(m!/2)4^{m+1}$ and $(4 + 4m)(m!/2)4^{m+1}$. These are larger than previously known codes for the same PMEPR bounds.**

***Index Terms*—Golay complementary sequences, orthogonal frequency-division multiplexing (OFDM), Reed–Muller codes.**

I. Introduction

In a fundamental paper [2], Golay presented the construction of complementary binary sequences. These sequences have found numerous applications in various fields of science and engineering. An important application of Golay complementary sequences is to orthogonal frequency-division multiplexing (OFDM). This is a communication technique with a long history which is rapidly emerging as a technology of choice in wireless applications. International standards such as IEEE 802.11 are employing OFDM for wireless local-area network (LAN) applications. For wireless applications, an OFDM-based system can be of particular interest because it provides a greater immunity to impulse noise and fast fades and eliminates the need for equalizers, while efficient hardware implementations can be realized using fast Fourier transform (FFT) techniques.

One of the major impediments to deploying OFDM is the high peak-to-mean envelope power ratio (PMEPR) of uncoded OFDM signals. To prevent spectral growth of the OFDM signal in the form of intermodulation among subcarriers and out-of-band radiation, the transmit amplifier must be operated in its linear region. Amplifiers with large linear range are expensive and this can increase the cost of the implementation of OFDM. Moreover, if the peak transmit power is limited, either by regulatory or application constraints, then a high PMEPR has the effect of reducing the average power allowed under OFDM relative to that under constant power modulation techniques. This, in turn, reduces the range of OFDM transmissions. A number of approaches have been proposed to deal with this power control problem [1], [3], [7], [8], [11], [13].

A convenient approach to PMEPR reduction in OFDM transmission is to use codes constructed from Golay complementary sequences. These sequences are employed as pilot sequences by the European Telecommunications Standards Institute (ETSI) Broadband Radio Access Networks (BRAN) committee. These classes of sequences enjoy PMEPR as low as 2 and are very attractive for this reason [5], [6], [10], [9], [12]. In [6], Davis and Jedwab developed a powerful theory which yields 2^b-PSK Golay sequences as unions of cosets of the classical Reed–Muller codes and new generalizations of them. Special realization of Golay sequences as cosets of these codes were also given in [9], [10]. The underlying theory was further developed in [12].

Given the practical applications of 2^b-PSK Golay sequences and given that quadrature amplitude modulation (QAM) sequences are widely used in OFDM, it is natural to look for Golay complementary sequences with symbols chosen from 16-QAM and higher QAM constellations. The underlying theory for these sequences is highly underdeveloped. In this paper, we make a contribution in this direction by presenting a construction of 16-QAM Golay sequences of length $n = 2^m$ from quaternary phase-shift keying (QPSK) complementary sequences. We construct a total of $(14 + 12m)(m!/2)4^{m+1}$ sequences whose average envelope power is n. Of these, $(m!/2)4^{m+1}$ sequences have peak envelope power (PEP) bounded above by $3.6n$, three subsets of size $(4 + 4m)(m!/2)4^{m+1}$ contain sequences having PEPs less than of $2.8n$, $2.0n$, and $1.2n$, respectively, and a subset of size $(m!/2)4^{m+1}$ contains sequences that has PEP bounded above by $0.4n$. By selectively discarding some of these sequences, we obtain two codes having PMEPRs of 2.0 and 2.8. The constructed sequences are ideal for application as pilot sequences in future OFDM systems.

The outline of this paper is as follows. In Section II, we review some background material for the results developed in this paper. In Section III, we examine a construction of 16-QAM Golay sequences that was previously given in [4]. This construction yields two codes with PMEPR bounds of 2.0 and 3.6. We compute sizes of codes resulting from the construction. We then provide a new construction of 16-QAM Golay sequences and compute PEP bounds for the sequences. These sequences result in two codes: a code with PMEPR ≤ 2.0 that improves on the construction in [4] and new code with PMEPR ≤ 2.8. Finally, we compute code rates for these codes. In Section IV, we demonstrate that our technique automatically allows us to construct 8-QAM Golay sequences as a special case.

Manuscript received July 1, 2001; revised June 30, 2003. This work was supported in part by the National Science Foundation under the Alan T. Waterman Award, Grant CCR-0139398.

C. V. Chong was with the Division of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138 USA. He is now with the Economic Development Board, Singapore 679637, Singapore.

R. Venkataramani and V. Tarokh are with the Division of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138 USA (e-mail: ramanv@ieee.org).

Communicated by A. M. Klapper, Associate Editor for Sequences.

Digital Object Identifier 10.1109/TIT.2003.818418

II. Preliminaries

The transmitted OFDM signal is the real part of the complex signal

$$S(t)=\sum_{i=0}^{n-1} a_i(t)e^{2j\pi f_i t}$$

where f_i is the frequency of the ith carrier, $j=\sqrt{-1}$, and $a_i(t)$ is constant over a symbol period of duration T. To ensure orthogonality, the carrier frequencies are related by

$$f_i = f_0 + i\Delta f \tag{1}$$

where f_0 is the smallest carrier frequency, and Δf is an integer multiple of the OFDM symbol rate, i.e., $T\Delta f \in \mathbb{Z}$. Suppose $a_i(t)$ takes the value a_i over a given symbol period. Then, the corresponding OFDM signal (denoted by $S_{\boldsymbol{a}}(t)$) is given by

$$S_{\boldsymbol{a}}(t)=\sum_{i=0}^{n-1} a_i e^{2j\pi f_i t}. \tag{2}$$

The *instantaneous envelope power* of $S_{\boldsymbol{a}}(t)$ associated with the sequence $\boldsymbol{a}=(a_0,a_1,\dots,a_{n-1})$ is defined as $P_{\boldsymbol{a}}(t)=|S_{\boldsymbol{a}}(t)|^2$. Then, (1) and (2) imply that

$$\begin{aligned}P_{\boldsymbol{a}}(t) =& |S_{\boldsymbol{a}}(t)|^2\\ =&\left(\sum_{i=0}^{n-1} a_i e^{2j\pi f_i t}\right)\left(\sum_{k=0}^{n-1} a_k^* e^{-2j\pi f_k t}\right)\\ =&\sum_{i=0}^{n-1}\sum_{k=0}^{n-1} a_i a_k^* e^{2j\pi (i-k)\Delta f t}.\end{aligned} \tag{3}$$

Thus, the mean power of $S_{\boldsymbol{a}}(t)$ during a symbol period is

$$\frac{1}{T}\int_{[0,T]} P_{\boldsymbol{a}}(t)dt = ||\boldsymbol{a}||^2 \overset{\text{def}}{=} \sum_{k=0}^{n-1}|a_k|^2.$$

The PEP of a codeword is defined as

$$\text{PEP}(\boldsymbol{a})=\sup_{t\in[0,T]} P_{\boldsymbol{a}}(t)$$

and the PMEPR of a code is

$$\text{PMEPR}(\mathcal{C})=\max_{\boldsymbol{a}\in\mathcal{C}} \text{PEP}(\boldsymbol{a})/P_{\text{av}}(\mathcal{C})$$

where $P_{\text{av}}(\mathcal{C})$ is the mean envelope power of an OFDM signal averaged over all OFDM signals generated from a codebook $\mathcal{C}$, i.e.,

$$\begin{aligned}P_{\text{av}}(\mathcal{C}) =& \frac{1}{T}\sum_{\boldsymbol{a}\in\mathcal{C}} p(\boldsymbol{a})\int_{[0,T]} P_{\boldsymbol{a}}(t)dt\\ =& \frac{1}{T}\sum_{\boldsymbol{a}\in\mathcal{C}} p(\boldsymbol{a})||\boldsymbol{a}||^2\end{aligned}$$

where $p(\boldsymbol{a})$ is the probability of transmitting the codeword $\boldsymbol{a}$.

Letting $k=i+u$ in (3) we obtain

$$\begin{aligned}P_{\boldsymbol{a}}(t) =& \sum_u\sum_i a_i a_{i+u}^* e^{2j\pi u\Delta f t}\\ =& \sum_{i=0}^{n-1}|a_i|^2+\sum_{u\neq 0}\sum_i a_i a_{i+u}^* e^{2j\pi u\Delta f t}\end{aligned} \tag{4}$$

where the summations are performed indexes i and u for which both i and $i+u$ lie in $\{0,1,\dots,n-1\}$. Let the aperiodic autocorrelation of sequence $\boldsymbol{a}$ at delay shift u be

$$C_{\boldsymbol{a}}(u)=\sum_i a_i a_{i+u}^*. \tag{5}$$

Then, $C_{\boldsymbol{a}}(0)=||\boldsymbol{a}||^2$. We can rewrite (4) as

$$P_{\boldsymbol{a}}(t)=||\boldsymbol{a}||^2+\sum_{u\neq 0} C_{\boldsymbol{a}}(u)e^{2j\pi u\Delta f t}. \tag{6}$$

Suppose that

$$\boldsymbol{x}=(x_0,x_1,\dots,x_{n-1})$$

and

$$\boldsymbol{y}=(y_0,y_1,\dots,y_{n-1})$$

are two complex-valued sequences of length n satisfying

$$C_{\boldsymbol{x}}(u)+C_{\boldsymbol{y}}(u)=(||\boldsymbol{x}||^2+||\boldsymbol{y}||^2)\delta(u) \tag{7}$$

where $\delta(u)=1$ if $u=0$ and $\delta(u)=0$ otherwise. Then, $\boldsymbol{x}$ and $\boldsymbol{y}$ are called *Golay complementary sequences*, named after Marcel J. E. Golay in recognition of his extensive study of their properties [2]. In the context of finite fields, we say that two sequences $\boldsymbol{x},\boldsymbol{y}\in\mathbb{Z}_H^n$ are *Golay complementary over* $\mathbb{Z}_H$ if the complex-valued sequences

$$(\xi^{x_1},\xi^{x_2},\dots,\xi^{x_n}) \quad \text{and} \quad (\xi^{y_1},\xi^{y_2},\dots,\xi^{y_n})$$

are Golay complementary, where $\xi=e^{j2\pi/H}$ is a primitive Hth root of unity. Finally, we say that $\boldsymbol{x}$ is a *Golay sequence* if there exists a sequence $\boldsymbol{y}$ that is complementary to $\boldsymbol{x}$. Golay made the following observation.

Lemma 1: Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be Golay complementary sequences of length n. Then, the peak value of $P_{\boldsymbol{a}}(t)$ is at most $||\boldsymbol{a}||^2+||\boldsymbol{b}||^2$.

Proof: Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be a Golay complementary pair, so that by definition $C_{\boldsymbol{a}}(u)+C_{\boldsymbol{b}}(u)=0$ for all $u\neq 0$. Then from (6), $P_{\boldsymbol{a}}(t)+P_{\boldsymbol{b}}(t)=||\boldsymbol{a}||^2+||\boldsymbol{b}||^2$, and since $P_{\boldsymbol{b}}(t)=|s_b(t)|^2\geq 0$, we deduce that $P_{\boldsymbol{a}}(t)\leq||\boldsymbol{a}||^2+||\boldsymbol{b}||^2$. □

Corollary 1: For constellations in which all symbols have unit power, any code $\mathcal{C}$ constructed from Golay sequences has $\text{PMEPR}(\mathcal{C})\leq 2$.

Proof: Note that all sequences in the codebook $\mathcal{C}$ have power n because all symbols in the constellation have unit power. Therefore, $||\boldsymbol{a}||^2=||\boldsymbol{b}||^2=P_{\text{av}}(\mathcal{C})=n$. Lemma 1 now implies that $\text{PEP}(\mathcal{C})\leq 2n$ and $\text{PMEPR}(\mathcal{C})\leq 2$. □

A. Reed–Muller Codes and Golay Sequences

We briefly review a result of Davis and Jedwab [6] that links a subset of 2^h-PSK Golay complementary sequences to first-order cosets of second-order Reed–Muller codes.

We confine the sequence length n to be a power of 2, i.e., $n=2^m$. Let $\underline{x}=(x_1,x_2,\dots,x_m)$ denote a vector of size m. A *generalized Boolean function* is a function u from $\mathbb{Z}_2^m=\{\underline{x}|x_i\in\{0,1\}\}$ to $\mathbb{Z}_2^h$ for an integer $h\geq 1$. A generalized Boolean function in m variables can be written in algebraic normal form as the sum of constant function 1 (zeroth-order monomial) and the rth-order monomials of the form $x_{j_1},x_{j_2},\dots,x_{j_r}$ for $1\leq r\leq m$, where $1\leq j_1,\dots,j_r\leq m$ are distinct numbers.

For any generalized Boolean function u in m variables, we can identify a $\mathbb{Z}_2$-valued vector $\boldsymbol{u} = (u_0, u_1, \ldots, u_{2^m-1})$ of length 2^m in which $u_i = u(\underline{i})$ where $\underline{i} = (i_1, i_2, \ldots i_m)$ is the binary expansion of i, i.e.,

$$i = \sum_{k=1}^{m} i_k 2^{m-k}.$$

The rth-order Reed–Muller code $\mathrm{RM}_{2^h}(r, m)$ over $\mathbb{Z}_{2^h}$ is a linear code of length 2^m generated by the monomials in the x_i of degree at most r. Davis and Jedwab [6] proved the following result.

Theorem 2: Let π denote any permutation of the set $\{1, 2, \ldots, m\}$, and $c_k \in \mathbb{Z}_{2^h}$. Then, the sequences generated by

$$u(\underline{x}) = 2^{h-1} \sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)} + \sum_{k=1}^{m} c_k x_k + c \qquad (8)$$

$$v(\underline{x}) = u(\underline{x}) + 2^{h-1} x_{\pi(\mu)} + c' \qquad (9)$$

are Golay complementary over $\mathbb{Z}_{2^h}$ for any $c_k, c, c' \in \mathbb{Z}_{2^h}$ and $\mu \in \{1, m\}$

Let $\mathcal{U}$ denote the set of Golay sequences of the form (8), and $\mathcal{V}(\boldsymbol{u})$, the set of sequences $\boldsymbol{v}$ of the form (9) that are complementary to $\boldsymbol{u} \in \mathcal{U}$. Clearly

$$|\mathcal{U}| = (m!/2)\,(2^h)^{m+1} \quad \text{and } |\mathcal{V}(\boldsymbol{u})| = 2(2^h) \qquad (10)$$

for all $\boldsymbol{u} \in \mathcal{U}$, where $|\cdot|$ denotes the cardinality of a set. Thus, Theorem 2 provides a set of $m!(2^h)^{m+2}$ pairs of complementary sequences. Numerical searches were unable to find any other Golay sequences in $\mathbb{Z}_{2^h}$ of length 2^m, but it is unproved this construction produces all Golay sequences.

III. 16-QAM Golay Sequences

Rößing and Tarokh [4] demonstrated a construction of 16-QAM sequences from QPSK Golay complementary sequences, and derived bounds for the PMEPR of the 16-QAM sequences. They observed that any 16-QAM symbol can be decomposed uniquely into a pair of QPSK symbols because any point on the 16-QAM constellation can be written as

$$q(u, v) = \alpha e^{j\pi/4} \xi^u + \beta e^{j\pi/4} \xi^v \qquad (11)$$

where $u \in \mathbb{Z}_4$ describes the quadrant in which the symbol lies and $v \in \mathbb{Z}_4$ identifies the location of the symbol within the quadrant and $\alpha, \beta \in \mathbb{R}$. We call u and v the *major and minor coordinates*, and α and β the *major and minor radii*. Assuming that all the 16-QAM symbols are equiprobable, we require $\alpha = 2/\sqrt{5}$ and $\beta = 1/\sqrt{5}$ for the constellation to have unit average energy. This representation of a 16-QAM symbol in terms of two QPSK symbols is shown in Fig. 1.

Rößing and Tarokh [4] construct 16-QAM sequences starting from Golay QPSK sequences and provide bounds on their PEP. The following is a summary of their result in an equivalent form.

Theorem 3: For any $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{Z}_4^n$, let $\boldsymbol{s}$ denote the 16-QAM sequence $s_i = q(u_i, v_i)$.

a) If $\boldsymbol{u}$ and $\boldsymbol{v}$ are Golay complementary sequences, then $\boldsymbol{s}$ and $\boldsymbol{t}$ are Golay complementary where $t_i = q(u_i, v_i + 2)$, and $\mathrm{PEP}(\boldsymbol{s}) \leq 2n$.

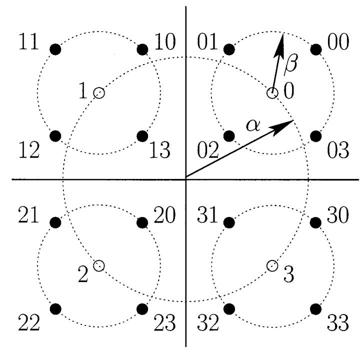

Fig. 1. Construction of 16-QAM symbols from two QPSK symbols.

b) If $\boldsymbol{u}$ and $\boldsymbol{v}$ are Golay sequences, but not necessarily complementary to each other, then $\mathrm{PEP}(\boldsymbol{s}) \leq 3.6n$.

Theorem 2, which provides an explicit construction of complementary sequences in $\mathbb{Z}_{2^h}$, can be used in conjunction with Theorem 3 to produce codes whose PMEPRs can be bounded. In the remainder of the paper, we assume that $h = 2$. Define two sets of sequences (codes) $\mathcal{C}_a$ and $\mathcal{C}_b$ as follows:

$$\mathcal{C}_a = \{\boldsymbol{s} : s_i = q(u_i, v_i),\ \boldsymbol{u} \in \mathcal{U}, \boldsymbol{v} \in \mathcal{V}(\boldsymbol{u})\}$$

$$\mathcal{C}_b = \{\boldsymbol{s} : s_i = q(u_i, u'_i),\ \boldsymbol{u}, \boldsymbol{u'} \in \mathcal{U}\}$$

where $\mathcal{U}$ and $\mathcal{V}$ are the sets defined immediately after Theorem 2. In other words, $\mathcal{C}_1$ consists of all 16-QAM sequences $s_i = q(u_i, v_i)$ where $\boldsymbol{u}$ and $\boldsymbol{v}$ are Golay complementary sequences of the form (8) and (9) and $\mathcal{C}_2$ consists of 16-QAM sequences $s_i = q(u_i, u'_i)$ where $\boldsymbol{u}$ and $\boldsymbol{u'}$ are two Golay sequences of the form (8).

It is easily verified that $P_{\mathrm{av}}(\mathcal{C}_a) = P_{\mathrm{av}}(\mathcal{C}_b) = n$. Therefore, by (10) and Theorem 3, we have

$$|\mathcal{C}_a| = (m!)4^{m+2} \qquad \text{and} \quad \mathrm{PMEPR}(\mathcal{C}_a) \leq 2 \qquad (12)$$

$$|\mathcal{C}_b| = \left((m!/2)\,4^{m+1}\right)^2 \qquad \text{and} \quad \mathrm{PMEPR}(\mathcal{C}_b) \leq 3.6. \qquad (13)$$

A. New Construction of 16-QAM Golay Sequences

We now present a new construction of 16-QAM sequences. Recall that a 16-QAM symbol can be represented in terms of two QPSK symbols (major and minor coordinates) as in (11). The following theorem describes the new construction.

Theorem 4: Suppose the major and minor coordinates of a 16-QAM sequences are of the form

$$A(\underline{x}) = 2 \sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)} + \sum_{k=1}^{m} c_k x_k + c$$

$$a(\underline{x}) = A(\underline{x}) + s(\underline{x})$$

for any $c_k,\ c \in \mathbb{Z}_4$. Then the 16-QAM sequence $\mathcal{A}_i = \alpha\gamma\xi^{A_i} + \beta\gamma\xi^{a_i}$, where $\alpha, \beta \in \mathbb{R}$ and $\gamma = e^{j\pi/4}$ is a Golay sequence for the following offsets:

$$s(\underline{x}) = \begin{cases} d_0 + d_1 x_{\pi(1)} \\ d_0 + d_1 x_{\pi(m)} \\ d_0 + d_1 x_{\pi(w)} + d_2 x_{\pi(w+1)}, & 1 \leq w \leq m-1, \\ & 2d_0 + d_1 + d_2 = 0 \end{cases}$$

where $d_0, d_1, d_2 \in \mathbb{Z}_4$.

Proof: We first establish our notation and derive some basic expressions that we will use in the proof. Let

$\underline{i} = (i_1, i_2, \ldots, i_m)$ denote the binary representation of i, i.e., $i = \sum_{k=1}^{m} i_k 2^{m-k}$. Let A_i, a_i, and s_i denote the ith elements of the sequences generated from $A(\underline{x})$, $a(\underline{x})$, and $s(\underline{x})$, respectively, in $\mathbb{Z}_4$. Therefore,

$$A_i = 2\sum_{k=1}^{m-1} i_{\pi(k)} i_{\pi(k+1)} + \sum_{k=1}^{m} c_k i_k + c \tag{14}$$
$$a_i = A_i + s_i. \tag{15}$$

Define functions

$$B(\underline{x}) = A(\underline{x}) + 2x_{\pi(1)}$$
$$b(\underline{x}) = a(\underline{x}) + 2x_{\pi(1)}.$$

Thus, the sequences they generate are

$$B_i = A_i + 2i_{\pi(1)} \tag{16}$$
$$b_i = a_i + 2i_{\pi(1)} = A_i + s_i + 2i_{\pi(1)}. \tag{17}$$

Theorem 2 states that the sequences $(\xi^{A_0}, \xi^{A_1}, \ldots, \xi^{A_{2^m-1}})$ and $(\xi^{B_0}, \xi^{B_1}, \ldots, \xi^{B_{2^m-1}})$ are complementary. Furthermore, if $s(\underline{x})$ is a polynomial of degree 1, i.e.,

$$s(\underline{x}) = d_0 + \sum_{k=1}^{m} d_k x_k \Longrightarrow s_i = s + \sum_{k=1}^{m} d_k i_k$$

then (14) and (15) imply that

$$a_i = A_i + s_i = \sum_{k=1}^{m-1} i_{\pi(k)} i_{\pi(k+1)} + \sum_{k=1}^{m} c'_k i_k + c'$$

where $c' = c + d_0$ and $c'_k = c_k + d_k$. Using Theorem 2 again, we conclude that $(\xi^{a_0}, \xi^{a_1}, \ldots, \xi^{a_{n-1}})$ and $(\xi^{b_0}, \xi^{b_1}, \ldots, \xi^{b_{n-1}})$ are complementary. Therefore,

$$\sum_{i=0}^{n-u-1} \left[\xi^{(A_i - A_{i+u})} + \xi^{(B_i - B_{i+u})}\right] = 0 \tag{18}$$
$$\sum_{i=0}^{n-u-1} \left[\xi^{(a_i - a_{i+u})} + \xi^{(b_i - b_{i+u})}\right] = 0 \tag{19}$$

for $u > 0$. Note that $C(u)$ is conjugate-symmetric, i.e., $C(-u) = C^*(u)$, it suffices to study the properties of $C(u)$ for $u \geq 0$. We assume that $u \geq 0$ in the rest of the paper. Let

$$\boldsymbol{\mathcal{A}} = (\mathcal{A}_0, \mathcal{A}_1, \ldots, \mathcal{A}_{2^m-1}) \quad \text{and} \quad \boldsymbol{\mathcal{B}} = (\mathcal{B}_0, \mathcal{B}_1, \ldots, \mathcal{B}_{2^m-1})$$

be 16-QAM sequences defined as

$$\mathcal{A}_i = \alpha\gamma\xi^{A_i} + \beta\gamma\xi^{a_i}$$
$$\mathcal{B}_i = \alpha\gamma\xi^{B_i} + \beta\gamma\xi^{b_i}$$

where α and β are the major and minor radii of the QPSK components, $\xi = e^{j\pi/2} = j$ and $\gamma = e^{j\pi/4}$. We now prove that the sequences $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{B}}$ are complementary for specific choices of $s(\underline{x})$ stated in the theorem which are all polynomials of degree 1. Since $\alpha, \beta \in \mathbb{R}$ and $|\gamma|^2 = 1$, we obtain

$$C_{\mathcal{A}}(u) = \sum_{i=0}^{n-u-1} \left(\alpha\gamma\xi^{A_i} + \beta\gamma\xi^{a_i}\right)\left(\alpha\gamma\xi^{A_{i+u}} + \beta\gamma\xi^{a_{i+u}}\right)^*$$
$$= \sum_{i=0}^{n-u-1} \Big[\alpha^2\xi^{(A_i - A_{i+u})} + \beta^2\xi^{(a_i - a_{i+u})} + \alpha\beta\Big(\xi^{(A_i - a_{i+u})} + \xi^{(a_i - A_{i+u})}\Big)\Big]. \tag{20}$$

Similarly

$$C_{\mathcal{B}}(u) = \sum_{i=0}^{n-u-1} \Big[\alpha^2\xi^{(B_i - B_{i+u})} + \beta^2\xi^{(b_i - b_{i+u})} + \alpha\beta\Big(\xi^{(B_i - b_{i+u})} + \xi^{(b_i - B_{i+u})}\Big)\Big]. \tag{21}$$

Adding (20) and (21), and using (18) and (19), we obtain

$$C_{\mathcal{A}}(u) + C_{\mathcal{B}}(u) = \sum_{i=0}^{n-u-1} \alpha\beta\Big[\xi^{(A_i - a_{i+u})} + \xi^{(a_i - A_{i+u})} + \xi^{(B_i - b_{i+u})} + \xi^{(b_i - B_{i+u})}\Big] \tag{22}$$

for $u > 0$. Using (15)–(17) we obtain

$$\xi^{(A_i - a_{i+u})} + \xi^{(a_i - A_{i+u})} = \xi^{A_i - A_{i+u}}\left(\xi^{s_i} + \xi^{-s_{(i+u)}}\right)$$
$$\xi^{(B_i - b_{i+u})} + \xi^{(b_i - B_{i+u})} = \xi^{B_i - B_{i+u}}\left(\xi^{s_i} + \xi^{-s_{(i+u)}}\right)$$
$$= \xi^{A_i - A_{i+u}}\xi^{2i_{\pi(1)} - 2(i+u)_{\pi(1)}} \times \left(\xi^{s_i} + \xi^{-s_{(i+u)}}\right)$$

where $((i+u)_1, (i+u)_2, \ldots, (i+u)_m)$ is the binary representation of $(i+u)$. Summing the above equations over $0 \leq i \leq n-u-1$ and combining with (22) yields

$$C_{\mathcal{A}}(u) + C_{\mathcal{B}}(u) = \alpha\beta\sum_{i=0}^{n-u-1} \xi^{A_i - A_{i+u}}\left(\xi^{s_i} + \xi^{-s_{(i+u)}}\right) \times \left(1 + \xi^{2i_{\pi(1)} - 2(i+u)_{\pi(1)}}\right)$$
$$= \alpha\beta\sum_{i=0}^{n-u-1} \xi^{A_i - A_{i+u}}\left(\xi^{s_i} + \xi^{-s_{(i+u)}}\right) \times \left(1 + (-1)^{i_{\pi(1)} - (i+u)_{\pi(1)}}\right). \tag{23}$$

We need to verify that the above summation vanishes to prove that $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{B}}$ are complementary. Consider the following two cases.

Case 1: $s(\underline{x}) = d_0 + d_1 x_{\pi(w)} + d_2 x_{\pi(w+1)}$, where $1 \leq w \leq m-1$ and $d_0, d_1, d_2 \in \mathbb{Z}_4$ such that $2d_0 + d_1 + d_2 = 0$.

For a fixed $u > 0$, let $j = i + u$ have a binary representation $\underline{j} = (j_1, j_2, \ldots, j_m)$. Whenever $i_{\pi(1)} \neq j_{\pi_1}$, the corresponding term in (23) clearly vanishes. In the remaining terms, we have $i_{\pi(1)} = j_{\pi(1)}$. Let v denote the smallest index for which $i_{\pi(v)} \neq j_{\pi(v)}$, i.e., $v = \inf\{k : i_{\pi(k)} \neq j_{\pi(k)}\}$. This is guaranteed to exist because $j \neq i$. Furthermore $v \geq 2$. Let i' and j' denote indexes whose binary representations differ from those of i and j only at positions $\pi(k)$ for $k \leq v-1$, i.e.,

$$k \leq v-1 \Longrightarrow \quad i'_{\pi(k)} = 1 - i_{\pi(k)}, \quad j'_{\pi(k)} = 1 - j_{\pi(k)}$$
$$k \geq v \Longrightarrow \quad i'_{\pi(k)} = i_{\pi(k)}, \quad j'_{\pi(k)} = j_{\pi(k)}$$

Clearly, $j' = i' + u$. For $k \leq v-1$, we have $i_{\pi(k)} = j_{\pi(k)} \neq j'_{\pi(k)}$ while for $k = v$, we have $i_{\pi(v)} \neq j_{\pi(v)} = j'_{\pi(k)}$. Therefore,

$$i_{\pi(k)} + j'_{\pi(k)} = 1, \qquad k \leq v. \tag{24}$$

Recall that $s_i = d_0 + d_1 i_{\pi(w)} + d_2 i_{\pi(w+1)}$. Thus, whenever $w \geq v$, we have $s_i = s_{i'}$ and $s_j = s_{j'}$ because $i_{\pi(k)} = i'_{\pi(k)}$ and $j_{\pi(k)} = j'_{\pi(k)}$ for all $k \geq v$. For $w \leq v-1$, however, we use (24) to conclude that

$$s_i + s_{j'} = 2d_0 + d_1(i_{\pi(w)} + j'_{\pi(w)}) + d_2(i_{\pi(w+1)} + j'_{\pi(w+1)})$$

$$=2d_0+d_1+d_2.$$

Thus, if $2d_0+d_1+d_2=0$, we have $s_i=-s_{j'}$ and, similarly, $s_j=-s_{i'}$. Our last two observations imply that

$$\xi^{s_i}+\xi^{-s_j}=\xi^{s_{i'}}+\xi^{-s_{j'}}, \qquad 1\le w\le m-1. \tag{25}$$

Using (14) and the definition of v we see that

$$\begin{aligned}A_i-A_j=&2\sum_{k=1}^{m-1}(i_{\pi(k)}i_{\pi(k+1)}-j_{\pi(k)}j_{\pi(k+1)})\\&+\sum_{k=1}^{m}c_{\pi(k)}(i_{\pi(k)}-j_{\pi(k)})\\=&2\sum_{k=v-1}^{m-1}(i_{\pi(k)}i_{\pi(k+1)}-j_{\pi(k)}j_{\pi(k+1)})\\&+\sum_{k=v}^{m}c_{\pi(k)}(i_{\pi(k)}-j_{\pi(k)}).\end{aligned} \tag{26}$$

Similarly

$$\begin{aligned}A_{i'}-A_{j'}=2&\sum_{k=v-1}^{m-1}(i'_{\pi(k)}i'_{\pi(k+1)}-j'_{\pi(k)}j'_{\pi(k+1)})\\&+\sum_{k=v}^{m}c_{\pi(k)}(i'_{\pi(k)}-j'_{\pi(k)}).\end{aligned} \tag{27}$$

Subtracting (26) from (27) and using the definitions of i' and j' we obtain

$$\begin{aligned}(A_{i'}-A_{j'})-(A_i-A_j)=&2(i'_{\pi(v-1)}i'_{\pi(v)}-j'_{\pi(v-1)}j'_{\pi(v)})\\&-2(i_{\pi(v-1)}i_{\pi(v)}-j_{\pi(v-1)}j_{\pi(v)})\\=&2.\end{aligned}$$

Therefore,

$$\xi^{A_{i'}-A_{j'}}=-\xi^{A_i-A_j}. \tag{28}$$

In view of (25) and (28) it follows that replacing i by i' in (23) negates the whole summation. Since the mapping from $i\to i'$ is invertible, the summation (23) must vanish.

Case 2: $s(\underline{x})=d_0+d_1x_{\pi(m)}$ or $s(\underline{x})=d_0+d_1x_{\pi(1)}$, where $d_0,d_1\in\mathbb{Z}_4$.

First consider the case $s_i=d_0+d_1i_{\pi(m)}$. We define i', j', and v as in Case 1. Since $v-1<m$, we always have $s_i=s_{i'}$ and $s_j=s_{j'}$ with no additional conditions on d_0 and d_1. Going through the proof as before, we conclude that (23) vanishes. Finally, using the permutation $\pi'(k)=\pi(m+1-k)$ instead of $\pi(k)$, we see that (23) also vanishes for the choice $s_i=d_0+d_1i_{\pi'(m)}=d_0+d_1i_{\pi(1)}$. □

Observe that the major and minor sequences are complementary in $\mathbb{Z}_4$ for offsets of the form $s(\underline{x})=d_0+2x_{\pi(1)}$ and $s(\underline{x})=d_0+2x_{\pi(m)}$. Thus, all the sequences in case a) of Theorem 3 are included in the above construction.

Corollary 5: Theorem 4 yields $(14+12m)(m!/2)4^{m+1}$ 16-QAM Golay sequences for $m\ge 2$.

Proof: The following is a list of all the *distinct* offset polynomials $s(\underline{x})$:

a) $s(\underline{x})=d_0$;
b) $s(\underline{x})=d_0+d_1x_{\pi(1)}$, for $d_1\neq 0$;
c) $s(\underline{x})=d_0+d_1x_{\pi(m)}$, for $d_1\neq 0$;
d) $s(\underline{x})=d_0+d_1x_{\pi(w)}$, for $2d_0+d_1=0$, $d_1\neq 0$, and $2\le w\le m-1$;
e) $s(\underline{x})=d_0+d_1x_{\pi(w)}+d_2x_{\pi(w+1)}$ for $2d_0+d_1+d_2=0$, $d_1\neq 0$, $d_2\neq 0$, and $1\le w\le m-1$ where $d_0,d_1,d_2\in\mathbb{Z}_4$.

We find that this is a collection of $(14+12m)$ distinct polynomials. The number of major sequences is clearly $(m!/2)4^{m+1}$. Therefore, the construction of Theorem 4 produces a total of $(14+12m)(m!/2)4^{m+1}$ 16-QAM sequences. □

B. PEP Bounds

We now compute PEP upper bounds for the 16-QAM Golay sequences constructed in Theorem 4. Recall that the ith symbol of a transmitted 16-QAM sequence is given by

$$\mathcal{A}_i=q(A_i,a_i)=\gamma(\alpha\xi^{A_i}+\beta\xi^{a_i})$$

where $\alpha=2/\sqrt{5}$, $\beta=1/\sqrt{5}$, $\gamma=e^{j\pi/4}$, and $a_i=A_i+s_i$. Therefore,

$$\begin{aligned}|\mathcal{A}_i|^2=&|\gamma(\alpha\xi^{A_i}+\beta\xi^{a_i})|^2=|\alpha+\beta\xi^{s_i}|^2\\=&\begin{cases}|\alpha+\beta|^2=1.8, & \text{if } s_i=0\\ |\alpha|^2+|\beta|^2=1, & \text{if } s_i\in\{1,3\}\\ |\alpha-\beta|^2=0.2, & \text{if } s_i=2.\end{cases}\end{aligned}$$

Therefore, the energy of the sequence $\boldsymbol{\mathcal{A}}$ is

$$\begin{aligned}\|\boldsymbol{\mathcal{A}}\|^2=&\sum_{i=0}^{2^m-1}\|\mathcal{A}_i\|^2\\=&1.8n_0(\boldsymbol{\mathcal{A}})+n_1(\boldsymbol{\mathcal{A}})+0.2n_2(\boldsymbol{\mathcal{A}})+n_3(\boldsymbol{\mathcal{A}})\end{aligned}$$

where $n_s(\boldsymbol{\mathcal{A}})$ is the number of times the symbol s occurs in the sequence $\boldsymbol{s}=\boldsymbol{\mathcal{A}}-\boldsymbol{a}$.

From the proof of Theorem 4 we see that $\boldsymbol{\mathcal{A}}$ is complementary to $\boldsymbol{\mathcal{B}}$ where $\mathcal{B}_i$ is either $\mathcal{A}_i(-1)^{i_{\pi(1)}}$ or $\mathcal{A}_i(-1)^{i_{\pi(m)}}$. In either case, $\|\boldsymbol{\mathcal{A}}\|^2=\|\boldsymbol{\mathcal{B}}\|^2$. Hence, by Lemma 1

$$\begin{aligned}\text{PEP}(\boldsymbol{\mathcal{A}})\le&\|\boldsymbol{\mathcal{A}}\|^2+\|\boldsymbol{\mathcal{B}}\|^2=2\|\boldsymbol{\mathcal{A}}\|^2\\=&3.6n_0(\boldsymbol{\mathcal{A}})+2n_1(\boldsymbol{\mathcal{A}})\\&+0.4n_2(\boldsymbol{\mathcal{A}})+2n_3(\boldsymbol{\mathcal{A}}).\end{aligned} \tag{29}$$

Tables I and II list the PEP bound (29) for each of the $(14+12m)$ offsets $s(\underline{x})$ in the construction of the 16-QAM sequences.

Our construction gives us complementary 16-QAM sequences whose PMEPR bounds are easily calculated. For each $\sigma\in\{0.4,1.2,2.0,2.8,3.6\}$, let $\mathcal{C}_\sigma$ denote the collection of 16-QAM sequences that correspond to a PEP bound of σn. Since the energy of each Golay sequences constructed is one half of its PEP bound (see (29)), we have

$$\|\boldsymbol{\mathcal{A}}\|^2=\sigma n/2, \qquad \forall\boldsymbol{\mathcal{A}}\in\mathcal{C}_\sigma.$$

Therefore, $P_{ac}(\mathcal{C}_{2.0})=n$ and when viewed as a code, $\mathcal{C}_{2.0}$ has the following characteristics:

$$|\mathcal{C}_{2.0}|=(2+2m)m!4^{m+1} \quad \text{and} \quad \text{PMEPR}(\mathcal{C}_{2.0})=2.0. \tag{30}$$

From (12) and (30) we see that $\mathcal{C}_{2.0}$ and $\mathcal{C}_a$ have the same PMEPR of 2.0, but $\mathcal{C}_{2.0}$ has a larger code rate.

Define another code $\mathcal{C}_c=\mathcal{C}_{1.2}\cup\mathcal{C}_{2.0}\cup\mathcal{C}_{2.8}$. Then, $P_{\text{av}}(\mathcal{C}_c)=n$, and we obtain

$$|\mathcal{C}_c|=(6+6m)m!4^{m+1} \quad \text{and} \quad \text{PMEPR}(\mathcal{C}_c)=2.8. \tag{31}$$

This is a new code with a larger code rate and a larger PMEPR than $\mathcal{C}_{2.0}$. Finally, we could define a code

$$\mathcal{C}_d=\mathcal{C}_{0.4}\cup\mathcal{C}_{1.2}\cup\mathcal{C}_{2.0}\cup\mathcal{C}_{2.8}\cup\mathcal{C}_{3.6}$$

TABLE I
PEP BOUNDS FOR CONSTRUCTED 16-QAM SEQUENCES OF LENGTH $n = 2^m$

$s(\underline{x})$	Possibilities	$n_0(\mathcal{A})$	$n_1(\mathcal{A})$	$n_2(\mathcal{A})$	$n_3(\mathcal{A})$	PEP bound
$\mathbf{0}$	1	n	0	0	0	$3.6n$
$x_{\pi(1)}$	1	$n/2$	$n/2$	0	0	$2.8n$
$x_{\pi(m)}$	1	$n/2$	$n/2$	0	0	$2.8n$
$2x_{\pi(1)}$	1	$n/2$	0	$n/2$	0	$2.0n$
$2x_{\pi(m)}$	1	$n/2$	0	$n/2$	0	$2.0n$
$3x_{\pi(1)}$	1	$n/2$	0	0	$n/2$	$2.8n$
$3x_{\pi(m)}$	1	$n/2$	0	0	$n/2$	$2.8n$
$x_{\pi(w)} + 3x_{\pi(w+1)}$	$m-1$	$n/2$	$n/4$	0	$n/4$	$2.8n$
$2x_{\pi(w)} + 2x_{\pi(w+1)}$	$m-1$	$n/2$	0	$n/2$	0	$2.0n$
$3x_{\pi(w)} + x_{\pi(w+1)}$	$m-1$	$n/2$	$n/4$	0	$n/4$	$2.8n$
$\mathbf{1}$	1	0	n	0	0	$2.0n$
$\mathbf{1} + x_{\pi(1)}$	1	0	$n/2$	$n/2$	0	$1.2n$
$\mathbf{1} + x_{\pi(m)}$	1	0	$n/2$	$n/2$	0	$1.2n$
$\mathbf{1} + 3x_{\pi(1)}$	1	$n/2$	$n/2$	0	0	$2.8n$
$\mathbf{1} + 3x_{\pi(m)}$	1	$n/2$	$n/2$	0	0	$2.8n$
$\mathbf{1} + 2x_{\pi(w)}$	m	0	$n/2$	0	$n/2$	$2.0n$
$\mathbf{1} + x_{\pi(w)} + x_{\pi(w+1)}$	$m-1$	0	$n/4$	$n/2$	$n/4$	$1.2n$
$\mathbf{1} + 3x_{\pi(w)} + 3x_{\pi(w+1)}$	$m-1$	$n/2$	$n/4$	0	$n/4$	$2.8n$
$\mathbf{2}$	1	0	0	n	0	$0.4n$
$\mathbf{2} + x_{\pi(1)}$	1	0	0	$n/2$	$n/2$	$1.2n$
$\mathbf{2} + x_{\pi(m)}$	1	0	0	$n/2$	$n/2$	$1.2n$
$\mathbf{2} + 2x_{\pi(1)}$	1	$n/2$	0	$n/2$	0	$2.0n$
$\mathbf{2} + 2x_{\pi(m)}$	1	$n/2$	0	$n/2$	0	$2.0n$
$\mathbf{2} + 3x_{\pi(1)}$	1	0	$n/2$	$n/2$	0	$1.2n$
$\mathbf{2} + 3x_{\pi(m)}$	1	0	$n/2$	$n/2$	0	$1.2n$
$\mathbf{2} + x_{\pi(w)} + 3x_{\pi(w+1)}$	$m-1$	0	$n/4$	$n/2$	$n/4$	$1.2n$
$\mathbf{2} + 2x_{\pi(w)} + 2x_{\pi(w+1)}$	$m-1$	$n/2$	0	$n/2$	0	$2.0n$
$\mathbf{2} + 3x_{\pi(w)} + x_{\pi(w+1)}$	$m-1$	0	$n/4$	$n/2$	$n/4$	$1.2n$
$\mathbf{3}$	1	0	0	0	n	$2.0n$
$\mathbf{3} + x_{\pi(1)}$	1	$n/2$	0	0	$n/2$	$2.8n$
$\mathbf{3} + x_{\pi(m)}$	1	$n/2$	0	0	$n/2$	$2.8n$
$\mathbf{3} + 3x_{\pi(1)}$	1	0	0	$n/2$	$n/2$	$1.2n$
$\mathbf{3} + 3x_{\pi(m)}$	1	0	0	$n/2$	$n/2$	$1.2n$
$\mathbf{3} + 2x_{\pi(w)}$	m	0	$n/2$	0	$n/2$	$2.0n$
$\mathbf{3} + x_{\pi(w)} + x_{\pi(w+1)}$	$m-1$	$n/2$	$n/4$	0	$n/4$	$2.8n$
$\mathbf{3} + 3x_{\pi(w)} + 3x_{\pi(w+1)}$	$m-1$	0	$n/4$	$n/2$	$n/4$	$1.2n$

TABLE II
NUMBER OF CONSTRUCTED 16-QAM GOLAY SEQUENCES OF LENGTH $n = 2^m$ FOR EACH OF THE PEP BOUNDS

PEP bound	Number of 16-QAM sequences
$3.6n$	$(m!/2)4^{m+1}$
$2.8n$	$(2+2m)m!4^{m+1}$
$2.0n$	$(2+2m)m!4^{m+1}$
$1.2n$	$(2+2m)m!4^{m+1}$
$0.4n$	$(m!/2)4^{m+1}$

TABLE III
CODE RATES OF CONSTRUCTED 16-QAM SEQUENCES

$n = 2^m$	$R(\mathcal{C}_{2.0})$	$R(\mathcal{C}_c)$
4	2.3962	2.7925
8	1.6981	1.8962
16	1.1192	1.2182
32	0.7029	0.7524
64	0.4266	0.4513
128	0.2523	0.2647
256	0.1464	0.1526
512	0.0836	0.0867
1024	0.0471	0.0487

which has a PMEPR of 3.6 and $|\mathcal{C}_d| = (7+6m)m!4^{m+1}$. However, owing to its smaller code size, this code is worse than the code $\mathcal{C}_b$ with characteristics (13).

C. Code Rates for Constructed Complementary 16-QAM Sequences

The code rate of a code $\mathcal{C}$ consisting of sequences of length n symbols is

$$R(\mathcal{C}) = \frac{\log_2 |\mathcal{C}|}{n} \text{ bits/ symbol.}$$

where $|\mathcal{C}|$ is the number of codewords. Table III shows the code rates for $\mathcal{C}_{2.0}$ and $\mathcal{C}_c$ for various values of $n = 2^m$. Observe that the code rates are reasonable for small m, but drop quickly for increasing lengths. These codes would be unsuitable for general use in OFDM systems with a large number (32 or more) of subcarriers. However, the constructed sequences could see practical application in the design of pilot symbols, as well as in scenarios where peak power control and encoder complexity are overriding concerns, not code rate. One such scenario could be in mobile handsets, where high peaks in the transmitted signal would have a deleterious effect on battery life.

IV. 8-QAM GOLAY SEQUENCES

Consider the 16-QAM Golay sequences constructed from two QPSK sequences described in Section III. Suppose we limit the possible values for the minor coordinates to 0 or 2, this results in the 8-QAM constellation is shown in Fig. 2. If the symbols are equiprobable, we need $\alpha = 2/\sqrt{5}$ and $\beta = 1/\sqrt{5}$ for the constellation to have unit average energy.

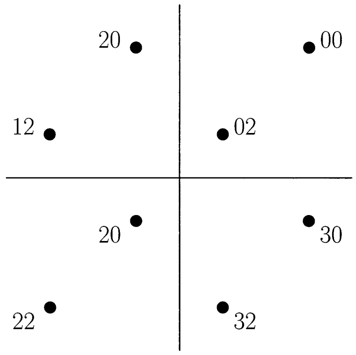

Fig. 2. 8-QAM signal constellation.

We can easily construct 8-QAM Golay sequences by restricting the minor sequence a_i to be even. Thus, the general form for $a(\underline{x})$ is

$$a(\underline{x}) = 2\sum_{k=1}^{m-1} x_{\pi(k)}x_{\pi(k+1)} + 2\sum_{k=1}^{m} c_k x_k + 2c$$

where $c_k \in \{0, 1\}$ and $c = 0$ or 1. The corresponding major sequence takes the form

$$A(\underline{x}) = a(\underline{x}) - s(\underline{x})$$

for any of the $14 + 12m$ polynomials $s(\underline{x})$ listed in Theorem 4. Note that the above statement is equivalent to the original statement of Theorem 4 because s is a polynomial of degree 1. This construction results in a total of $(14+12m)(m!/2)2^{m+1}$ Golay sequences. Of these, two sets of $(m!/2)2^{m+1}$ sequences have PEP bounds of $3.6n$ and $0.4n$, respectively, and three sets of $(4 + 4m)(m!/2)2^{m+1}$ sequences each having PEP bounds of $2.8n$, $2.0n$, and $1.2n$, respectively.

V. Conclusion

In this paper, we initiated the studies of 16- and 8-QAM Golay complementary sequences. We constructed a new class of 16-QAM Golay sequences consisting of $(14 + 12m)(m!/2)4^{m+1}$ sequences. When employed in an OFDM system, $(m!/2)4^{m+1}$ of these sequences have PEP bounded above by $3.6n$, three subsets of size $(4 + 4m)(m!/2)4^{m+1}$ contain sequences having PEPs of less than $2.8n$, $2.0n$, and $1.2n$, respectively, and a subset of size $(m!/2)4^{m+1}$ contains sequences that have PEP upper-bounded by $0.4n$. The average envelope power of all the sequences is n. These sequences are used to construct two codes of PMEPRs of 2.0 and 2.8. When restricted to 8-QAM, we obtain $(14 + 12m)(m!/2)2^{m+1}$ Golay sequences with two sets of $(m!/2)2^{m+1}$ sequences each having PEP bounds of $3.6n$ and $0.4n$, respectively, and three sets of $(4 + 4m)(m!/2)2^{m+1}$ sequences each having PEP bounds of $2.8n$, $2.0n$, and $1.2n$, respectively. Computer studies indicate that there other 8- and 16-QAM Golay complementary sequences than those we constructed here. Unfortunately, these sequences were difficult to classify based on their polynomial structure. The classification of these sequences and their realization as OFDM QAM codes with low encoding/decoding complexities remains an interesting open problem.

References

[1] M. Friese, "Multicarrier modulation with low peak-to-mean average power ratio," *Electron. Lett.*, vol. 32, pp. 713–714, 1996.

[2] M. J. E. Golay, "Complementary series," *IRE Trans. Inform. Theory*, vol. IT-7, pp. 82–87, Apr. 1961.

[3] T. F. Ho and V. K. Wei, "Synthesis of low-crest waveforms for multicarrier CDMA systems," *Proc. IEEE GLOBECOMM 1995*, pp. 131–135, 1995.

[4] C. Rößing and V. Tarokh, "A construction of OFDM 16-QAM sequences having low peak powers," *IEEE Trans. Inform. Theory*, vol. 47, pp. 2091–2094, Nov. 2001.

[5] J. A. Davis and J. Jedwab, "Peak-to-mean power control and error correction for OFDM transmission using Golay sequences and reed-muller codes," *Electron. Lett.*, vol. 33, pp. 267–268, 1997.

[6] ——, "Peak-to-mean power control in OFDM, Golay complementary sequences, and reed-muller codes," *IEEE Trans. Inform. Theory*, vol. 45, pp. 2397–2417, Nov. 1999.

[7] X. Li and L. J. Cimini Jr., "Effects of clipping and filtering on the performance of OFDM," *Proc. IEEE 47th Vehicular Technology Conf.*, pp. 1634–1638, 1997.

[8] S. H. Müller, R. W. Bäuml, R. F. H. Fischer, and J. B. Huber, "OFDM with reduced peak-to-average power ratio by multiple signal representation," *Ann. Télécommun.*, vol. 52, pp. 58–67, 1997.

[9] R. D. J. van Nee, "OFDM codes for peak-to-average power reduction and error correction," *Proc. IEEE GLOBECOM 1996*, pp. 740–744, 1996.

[10] H. Ochiai and H. Imai, "Block coding scheme based on complementary sequences for multicarrier signals," *IEICE Trans. Fundamentals*, pp. 2136–2143, 1997.

[11] R. O'Neill and L. B. Lopes, "Envelope variations and spectral splatter in clipped multicarrier signals," in *Proc. Conf. Personal Indoor and Mobile Radio Communications (PIMRC'95)*, Sept. 1995, pp. 71–75.

[12] K. G. Paterson, "Generalized reed-muller codes and power control in OFDM modulation," *IEEE Trans. Inform. Theory*, vol. 46, pp. 104–120, Jan 2000.

[13] D. Wulich, "Reduction of peak-to-mean ratio of multicarrier modulation using cyclic coding," *Electron. Lett.*, vol. 32, pp. 432–433, 1996.